4.2: Prime and Composite Numbers

Definition: A prime number is a number with exactly two distinct positive factors, namely 1 and themselves.

Definition: A composite number is a number with more than two distinct positive factors:

Is 1 a prime number or a composite number?

Find which numbers are prime in the set {1, 2, ..., 100}

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

This is known as the Sieve of Eratosthenes.

Theorem: If n is composite, then it has a prime factor p with the property that $p^2 \leq n$.

In other words, to see if a number is prime, we need only check all of the possible prime factors up to its square root.

Proof:

Example: List the factors of 28. Is 28 prime or composite?

Is 301 prime?

Is 307 prime?

Fundamental Theorem of Arithmetic: Each composite number can be written as a product of primes in exactly one way (ignoring the order of the factors).

Definition: This product described above is known as the prime factorization of a number.

Example: What is the prime factorization of 120?

Example: What is the prime factorization of 270?